Transport of quantum noise through random media

P. Lodahl and A. Lagendijk

Complex Photonic Systems, Department of Science and Technology and MESA+ Research Institute, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

(Dated: October 15, 2004)

Abstract

We present an experimental study of the propagation of quantum noise in a multiple scattering random medium. Both static and dynamic scattering measurements are performed: the total transmission of noise is related to the mean free path for scattering, while the noise frequency correlation function determines the diffusion constant. The quantum noise observables are found to scale markedly differently with scattering parameters compared to classical noise observables. The measurements are explained with a full quantum model of multiple scattering.

PACS numbers: 42.25.Dd, 42.50.Lc, 78.67.-n

Light propagation in static disordered photonic media is coherent. The coherence is preserved even after a very large number of scattering events. Coherent transport of light in a disordered medium is the basis for applications of wave scattering to enhance communication capacities [1], for acoustical and biomedical imaging, as well as for fundamental discoveries of intensity correlations, enhanced backscattering, and Anderson localization [2]. All these phenomena are captured by classical physics where for light the electromagnetic field is described by Maxwell's wave equation. In contrast, no experiments on multiple scattering have been carried out yet in the realm of quantum optics where a classical description of light is insufficient and effects of the quantized nature of the electromagnetic field are encountered. Pioneering theoretical work includes a study of the propagation of coherent [3] and squeezed light [4] through a random medium with gain, and the associated quantum noise limited information capacities in such random lasers [5]. The photon statistics of a random laser was recently measured [6], which confirmed the expectations for a laser, and is an instructive example of the independent information that can be extracted by quantum optical measures. Here we present measurements of the transport of quantum and classical noise through a passive multiple scattering medium.

Noise is inevitable in all measurements. The fundamental lower limit is determined by quantum mechanics through Heisenberg's uncertainty relation, and is referred to as the shot noise limit. In the particle description of quantum mechanics, the existence of optical shot noise directly proves that light is quantized [7]. Shot noise fluctuations scale proportional to the square root of the average number of photons (particle-like behavior) in contrast to classical fluctuations that scale linearly (wave-like behavior). The different scaling allows us to distinguish quantum noise from classical noise in an optical experiment. Shot noise is universal to systems consisting of quantized entities and, e.g., offers independent information about the conduction of electrons in mesoscopic conductors compared to standard conductance measurements. Electronic shot noise has especially proven superior for detecting the charge of quasi-particles [8].

In the current Letter we investigate the propagation of classical and quantum intensity noise of light through a multiple scattering, randomly ordered medium. Two different measurements are presented in the noise: total transmission and (short range) frequency correlations. The two measurements provide insight in static and dynamic transport of quantum noise in a random medium, respectively.

Multiple scattering of light forms a volume intensity speckle pattern of bright and dark spots that most conveniently can be described as a discrete number of conduction channels. We consider the intensity transmitted from an input channel a to an output channel b, $I^{ab}_{\omega}(t)$, which depends on time t and the optical frequency ω. The intensity is expanded as a mean value $\overline{I^{ab}_{\omega}}$ plus a fluctuating part $\delta I^{ab}_{\omega}(t)$ that describes the quantum noise. In a total transmission measurement we sum up all output channels, and we introduce the total transmitted intensity $I^{a}_{\omega}(t) = \sum_b I^{ab}_{\omega}(t)$. We define the noise transmission coefficient as

$$\mathcal{T}^{N}_{a}(\Omega) = \frac{\overline{|\delta I^{a}_{\omega}(\Omega)|^2}}{\overline{|\delta I^{in}_{\omega}(\Omega)|^2}} = \frac{\overline{|\sum_b \delta I^{ab}_{\omega}(\Omega)|^2}}{\overline{|\delta I^{in}_{\omega}(\Omega)|^2}}, \tag{1}$$

where the bars denote average over measurement time and Ω is the frequency (Fourier transform of t) that accounts for slowly varying intensity fluctuations of light. $\overline{|\delta I^{in}_{\omega}(\Omega)|^2}$ is the spectral density of the input noise of the light illuminating the sample. The frequency auto-correlation function of noise in a single speckle spot (channel b) is defined as

$$C^{N}_{ab}(\Delta\omega, \Omega) = \tag{2}$$
$$\frac{\left\langle\left\langle \overline{|\delta I^{ab}_{\omega}(\Omega)|^2} \times \overline{\left|\delta I^{ab}_{\omega+\Delta\omega}(\Omega)\right|^2} \right\rangle\right\rangle_{\omega} - \left\langle\left\langle \overline{|\delta I^{ab}_{\omega}(\Omega)|^2} \right\rangle\right\rangle^2_{\omega}}{\left\langle\left\langle \overline{|\delta I^{ab}_{\omega}(\Omega)|^2} \right\rangle\right\rangle^2_{\omega}},$$

where double brackets $\langle\langle\cdot\cdot\rangle\rangle_{\omega}$ denote ensemble average that in this case is obtained by averaging over the optical frequency ω. Here noise correlation functions are introduced for the first time, while previous efforts have centered on measuring intensity correlation functions [9].

The transmitted noise spectral densities can be calculated using a full quantum model for multiple scattering [3]. We relate the annihilation operator of the output electric field in channel b ($\hat{a}^{ab}_{\omega}$) to the input electric field in channel a ($\hat{a}^{a}_{\omega}$) through the relation

$$\hat{a}^{ab}_{\omega}(\Omega) = t^{ab}_{\omega}\hat{a}^{a}_{\omega}(\Omega) + \sum_{a' \neq a} t^{a'b}_{\omega}\hat{a}^{a'}_{\omega}(\Omega) + \sum_{b'} r^{b'b}_{\omega}\hat{a}^{b'}_{\omega}(\Omega), \tag{3}$$

where indices a and b label channels on the input and output side of the multiple scattering medium, respectively. $\hat{a}^{a'}_{\omega}(\Omega)$ and $\hat{a}^{b'}_{\omega}(\Omega)$ account for vacuum fluctuations in all open channels while $t^{a'b}_{\omega}$ and $r^{b'b}_{\omega}$ are electric field transmission and reflection coefficients. It is straightforward to derive that the noise spectral density in a single channel b is given by

$$\overline{|\delta I^{ab}_{\omega}(\Omega)|^2} = \tag{4}$$
$$\left|t^{ab}_{\omega}\right|^4 \left(\overline{|\delta I^{in}_{\omega}(\Omega)|^2} - \overline{|\delta I^{v}_{\omega}(\Omega)|^2}\right) + \left|t^{ab}_{\omega}\right|^2 \overline{|\delta I^{v}_{\omega}(\Omega)|^2},$$

where we have defined the intensity spectral densities

$$\overline{\left|\delta I_\omega^{in/ab}(\Omega)\right|^2} = \left\langle \left[\hat{I}_\omega^{in/ab}(\Omega)\right]^2 \right\rangle - \left\langle \hat{I}_\omega^{in/ab}(\Omega) \right\rangle^2, \tag{5a}$$

$$\overline{\left|\delta I_\omega^{v}(\Omega)\right|^2} = \overline{I_\omega^{in}} \left\langle \hat{a}_\omega^v(\Omega)\hat{a}_\omega^v(\Omega)^\dagger \right\rangle, \tag{5b}$$

and superscript v indicates vacuum contribution and the single brackets $\langle\cdot\cdot\rangle$ quantum mechanical expectation values. For light dominated by classical noise (in the following referred to as technical noise) we can neglect the vacuum contribution in Eq. (4) and arrive at

$$\mathcal{T}_{ab}^{TN} = \left|t_\omega^{ab}\right|^4. \tag{6}$$

With a shot noise (SN) limited input field, we have $\overline{\left|\delta I_\omega^{in}(\Omega)\right|^2} = \overline{\left|\delta I_\omega^{v}(\Omega)\right|^2}$ and consequently

$$\mathcal{T}_{ab}^{SN} = \left|t_\omega^{ab}\right|^2. \tag{7}$$

By adding up all output channels and averaging over disorder it can be shown that [10]

$$\langle\langle \mathcal{T}_a^{SN} \rangle\rangle = \frac{\ell}{L}, \tag{8a}$$

$$\langle\langle \mathcal{T}_a^{TN} \rangle\rangle = \frac{\ell^2}{L^2}, \tag{8b}$$

where ℓ is the transport mean free path, and L is the sample thickness. Here we have omitted contributions from universal conductance fluctuations [11] that are negligible for the experimental parameters used here. Consequently, we have shown that the total transmission of quantum and classical noise are expected to vary linearly and quadratically with the ratio of the mean free path to the sample thickness, respectively.

The noise auto-correlation function, as defined in Eq. (2), is a fourth-order intensity correlation function. For classical waves, the general scaling of a correlation function of order $2n$ is as the second order correlation function to the power n [12]. Consequently, the noise auto-correlation is expected to follow the squared second order intensity correlation function [13]

$$C_{ab}^N(\Delta\omega) = \left[\frac{2L^2\Delta\omega/D}{\cosh\left(\sqrt{2L^2\Delta\omega/D}\right) - \cos\left(\sqrt{2L^2\Delta\omega/D}\right)}\right]^2. \tag{9}$$

In the following we will demonstrate experimentally that this scaling is obeyed by classical noise while quantum noise turns out to decay slower with frequency.

The experimental setup is outlined in Fig. 1. A frequency tunable titanium-sapphire laser was used to probe the random multiple scattering samples. The laser's amplitude noise spectrum was found to be shot noise limited above ~ 1.5 MHz and dominated by technical noise at smaller frequencies, which was carefully checked by observing that the noise of the input beam scaled quadratically and linearly with intensity, respectively. The two regimes enable us to study simultaneously the transmission of classical and quantum noise. We used strongly scattering samples consisting of titania particles (refractive index 2.7) with size distribution $d = 220 \pm 70$ nm [14] deposited on a fused silica substrate. Two types of experiments were carried out: total transmission and speckle frequency correlation measurements. In the former experiment, the transmitted diffuse light was collected with an integrating sphere onto a sensitive silicon photodiode (detector D1 in Fig. 1). The intensity noise was recorded by measuring the spectral density of the photocurrent $|\delta i(\Omega)|^2$ with a spectrum analyzer. Thermal noise from the detector was subtracted in the measurements. As a reference measurement, we recorded the intensity noise without any sample inserted. In the speckle correlation measurements we recorded the intensity noise in a single speckle spot (detector D2 in Fig. 1) that was selected using a pinhole. We subsequently varied the frequency of the laser and recorded a frequency speckle pattern. In total 200 noise spectra were measured at equally spaced optical frequencies with a frequency step of about 0.5 THz. From the complete measurement series the auto-correlation function was calculated.

Figure 2 displays two measurements of the total transmission of noise through samples with different thicknesses. The total transmission was obtained by measuring relative to the noise spectrum recorded with the sample removed. Two frequency regimes are apparent in the data: below ~ 1 MHz and above ~ 1.5 MHz corresponding to the frequencies where the input laser light was dominated by technical noise and shot noise, respectively. Between these frequencies the noise was dominated by oscillations in the detector power supply and therefore abandoned in the analysis. We observe immediately that the total transmission of classical noise is significantly lower than the total transmission of quantum noise. Within each noise regime we can define frequency averaged total transmission coefficients $\mathcal{T}_a^{SN}$ for shot noise and $\mathcal{T}_a^{TN}$ for technical noise, i.e. we skip the index Ω in the following.

Figure 3 shows the measured inverse total transmission as a function of sample thickness for both shot noise and technical noise. For each sample thickness the total transmission was measured several times at different positions on the sample in order to obtain ensemble-averaged transmission coefficients $\langle\langle T\rangle\rangle$. The experimental data are fitted with the theoretical model given by Eqs. (8), and very good agreement is observed. From the fits we extract the transport mean free path ℓ, and obtain $\ell_{SN} = 1.19 \pm 0.33\,\mu$m from the shot noise measurements and $\ell_{TN} = 1.03 \pm 0.09\,\mu$m from the technical noise measurements. The two values agree to within the error-bars of the measurements. A proper account for the boundaries of the sample has been included by effectively extending the sample thickness with extrapolation lengths determined by Fresnel corrections [15].

In the speckle-frequency correlation measurements we again compare the behavior of classical and quantum noise. The complete shot noise data set is given in Fig. 4. We record the photocurrent spectral density with a spectrum analyzer while tuning the optical frequency of the excitation beam. Figure 4 displays a frequency speckle pattern that contains information about the dynamical transport of noise in the multiple scattering medium. From that we can investigate the quantum noise speckle correlations. We compute the auto-correlation functions defined in Eq. (2) for both technical noise and shot noise, and their decay with frequency are shown in Fig. 5. The quantum noise correlation function is seen to decay much slower with frequency than the classical noise correlation function, which shows that quantum frequency correlations are extend further than the classical counterparts.

The correlation measurements in Fig. 5 are compared to the theoretical prediction of Eq. (9). For the technical noise data, we obtain good agreement with theory using the known value of the sample thickness ($L = 18\,\mu$m) and a diffusion constant of $D = 35\,\mathrm{m}^2/\mathrm{s}$. The latter is consistent with time-resolved propagation experiments on similar samples [14]. The slight oscillations in the experimental data are attributed to the limited statistics of 200 measurement points corresponding to about 20 independent speckle spots.

The quantum noise data violate the classical scaling argument leading to Eq. (9). The quantum noise correlation decays as the square-root of the classical noise correlation function: $C^{SN}_{ab}(\Delta\omega) = \sqrt{C^{N}_{ab}(\Delta\omega)}$, which can be understood as due to cancellation of the quadratic term in Eq. (4) for shot noise. Excellent agreement with this predicted scaling (full curve in Fig. 5) is found for our experimental data.

As a side result our experiments demonstrate the robustness of shot noise in multiple scattering: no excess noise was observed due to scattering as opposed to what is expected for an amplifying medium [3, 4]. This also points to an important difference between shot noise in electronics and optics. In a disordered metal wire, electronic shot noise is corrupted by thermal noise on the scale of the electron-phonon scattering length [8]. On the contrary, optical shot noise prevails over distances much longer than the (elastic) scattering length.

We have studied the propagation of classical and quantum noise through a multiple scattering medium. Both static and dynamic measurements were carried out and compared to theory, which allowed extracting fundamental scattering properties of the medium. The quantum fluctuations were found to scale markedly different with scattering parameters compared to classical fluctuations, hence explicitly demonstrating the difference between particle-like and wave-like transmission.

We would like like to acknowledge Allard Mosk, Ivo Vellekoop, and Valentin Freilikher for valuable discussions and useful comments on the manuscript. This work is part of the research program of the "Stichting voor Fundamenteel Onderzoek der Materie (FOM)", which is financially supported by the "Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO)".

[1] A.L. Moustakas, H.U. Baranger, L. Balents, A.M. Sengupta, and S.H. Simon, Science **287**, 287 (2000).

[2] P. Sheng, *Introduction to wave scattering, localization, and mesoscopic phenomena* (Academic Press, San Diego, 1995).

[3] M. Patra and C.W.J. Beenakker, Phys. Rev. A **60**, 4059 (1999).

[4] M. Patra and C.W.J. Beenakker, Phys. Rev. A **61**, 063805 (2000).

[5] J. Tworzydło and C.W.J. Beenakker, Phys. Rev. Lett. **89**, 043902 (2002).

[6] H. Cao, Y. Ling, J.Y. Xu, C.Q. Cao, and P. Kumar, Phys. Rev. Lett. **86**, 4524 (2001).

[7] C.H. Henry and R.F. Kazarinov, Rev. Mod. Phys. **68**, 801 (1996).

[8] Ya.M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).

[9] A.Z. Genack, N. Garcia, and W. Polkosnik, Phys. Rev. Lett. **65**, 2129 (1990).

[10] P. Lodahl, A.P. Mosk, and A. Lagendijk, in preparation.

[11] S. Feng, C. Kane, P.A. Lee, and A.D. Stone, Phys. Rev. Lett. **61**, 834 (1988).

[12] E. Kogan, M. Kaveh, R. Baumgartner, and R. Berkovits, Phys. Rev. B **48**, 9404 (1993).

[13] R. Berkovits and S. Feng, Phys. Rep. **238**, 135 (1994).

[14] R.H.J. Kop, P. de Vries, R. Sprik, and A. Lagendijk, Phys. Rev. Lett. **79**, 4369 (1997).

[15] J.X. Zhu, D.J. Pine, and D.A. Weitz, Phys. Rev. A **44**, 3948 (1991).

Figures

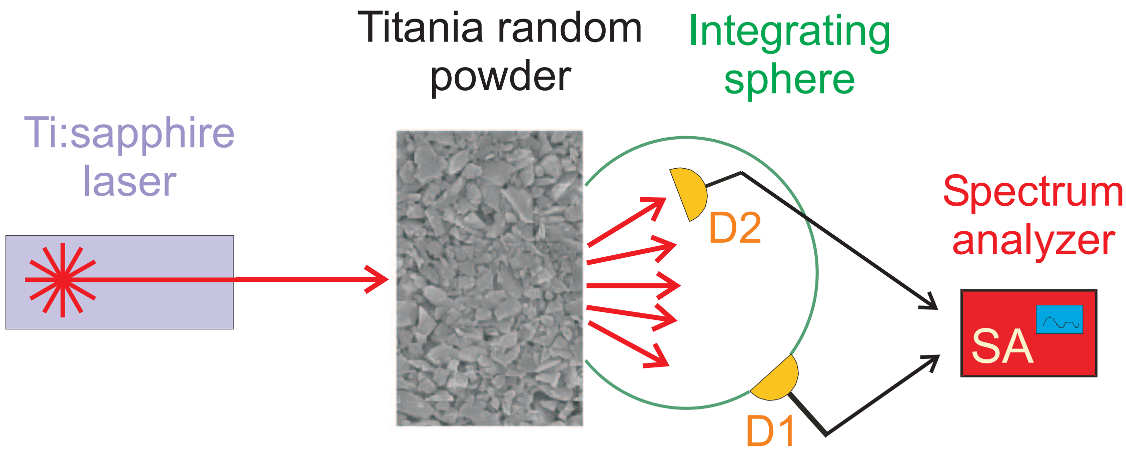

FIG. 1: Setup for measuring the transmission of quantum noise through a multiple scattering medium. Two different measurements were carried out by inserting either detector D1 or D2. The total transmission was recorded with an integrating sphere onto detector D1. With detector D2, the noise in a single speckle spot was measured.

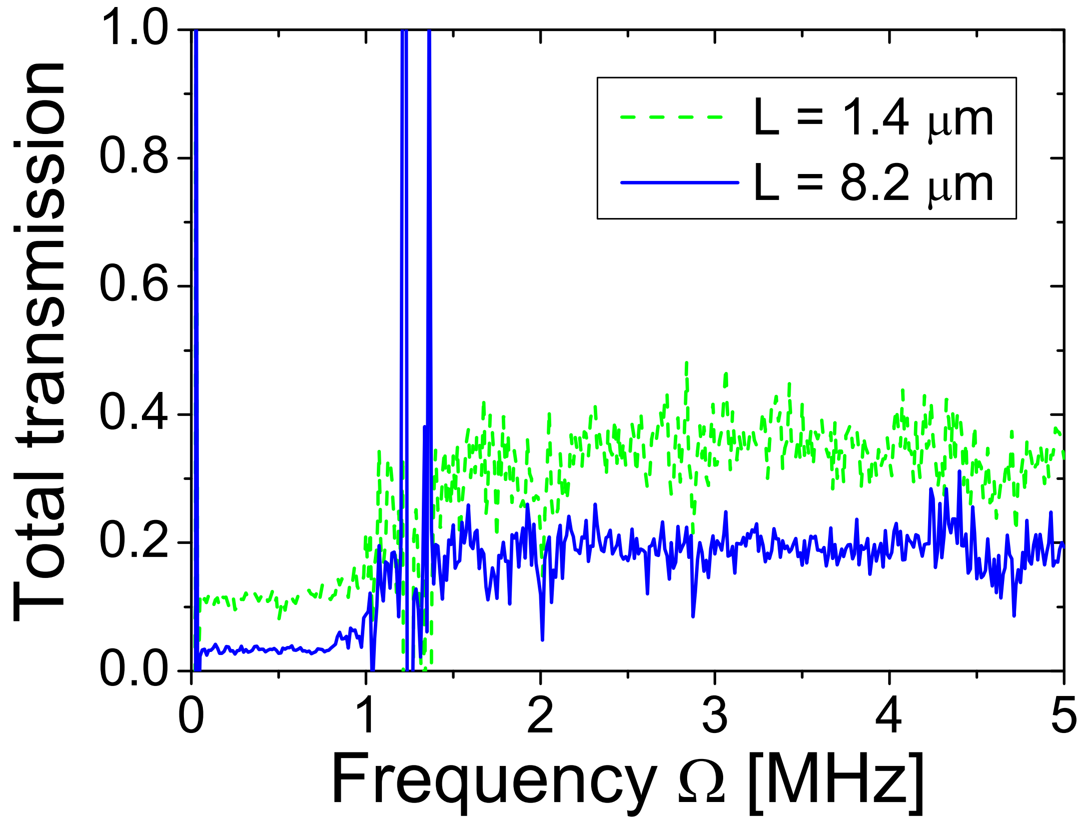

FIG. 2: Total transmission of noise as a function of measurement frequency Ω for two different sample thicknesses. The spectral densities were recorded with a resolution bandwidth of 30 kHz and a video bandwidth of 10 kHz and by averaging each trace 100 times. Radically different transmissions are observed for technical noise (below 1 MHz) compared to shot noise (above 1.5 MHz). The spikes around 1.3 MHz are due to oscillations in the detector power supply.

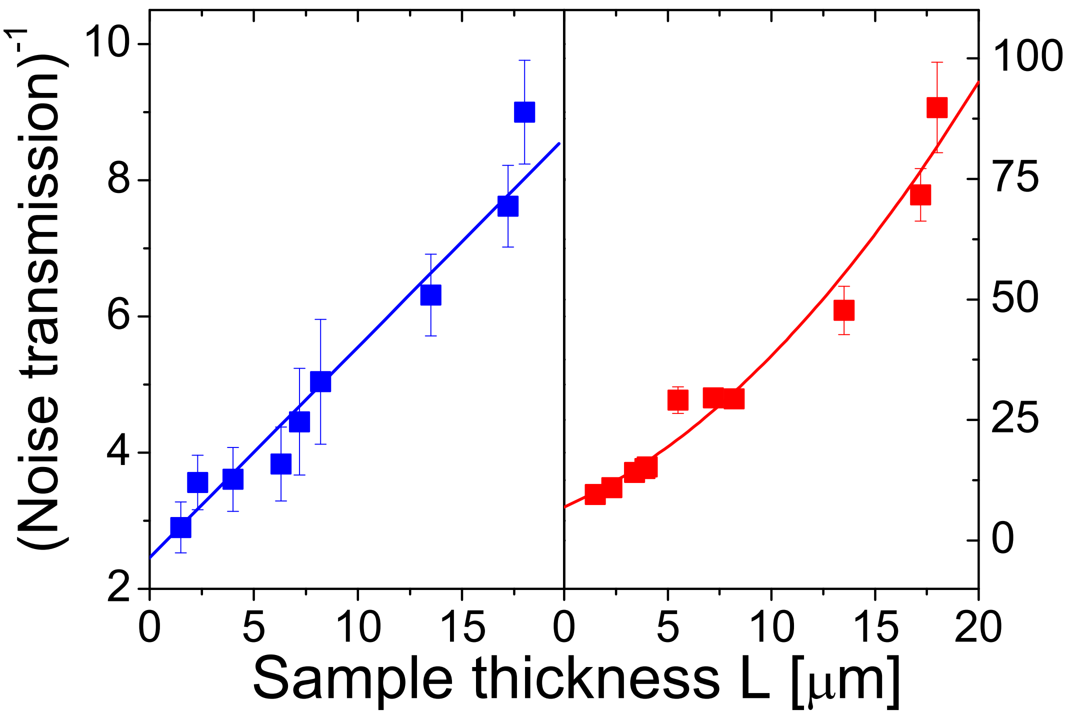

FIG. 3: Left panel: inverse total transmission of quantum noise as a function of sample thickness. The line is a linear fit to the experimental data. Right panel: inverse total transmission of technical noise and a quadratic fit to the data.

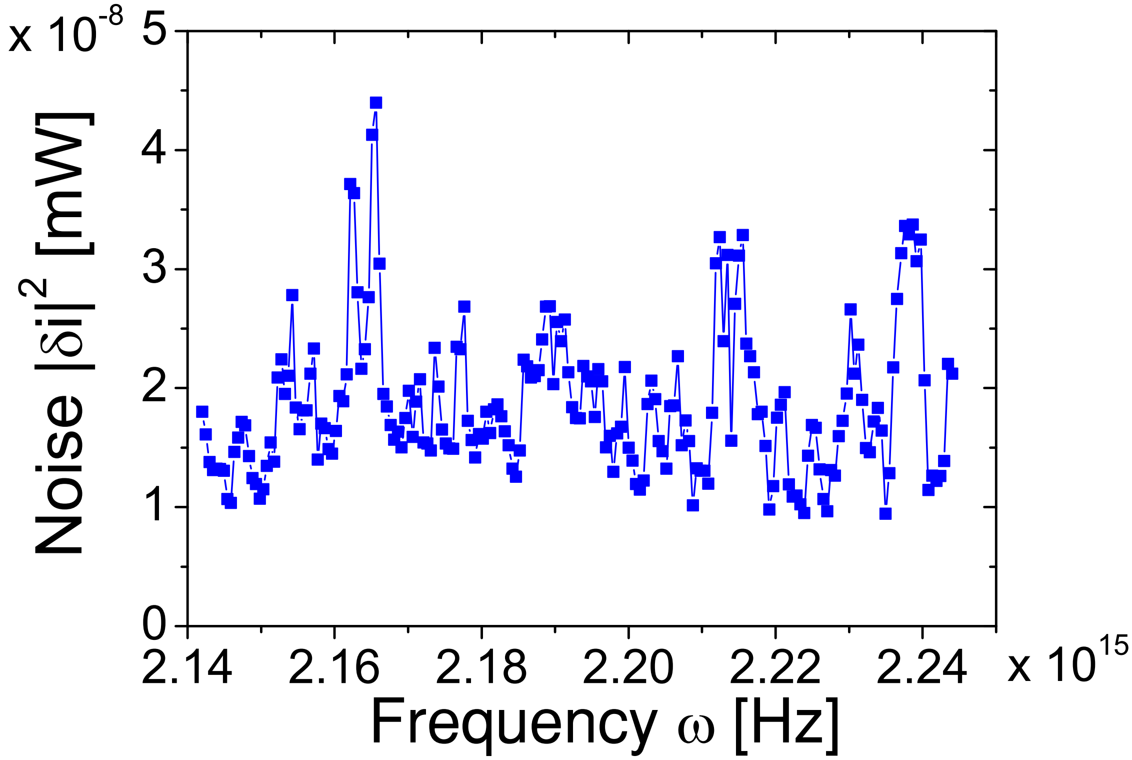

FIG. 4: Complete data set of measured photocurrent spectral density $|\delta i|^2$ in a single speckle spot for shot noise. The optical frequency ω of the input field was varied in steps of about 0.5 THz, and the noise spectra were recorded at in total 200 different optical frequencies. Each datapoint was obtained by averaging the noise spectra over the measurement frequency Ω within the limits of the shot noise region.

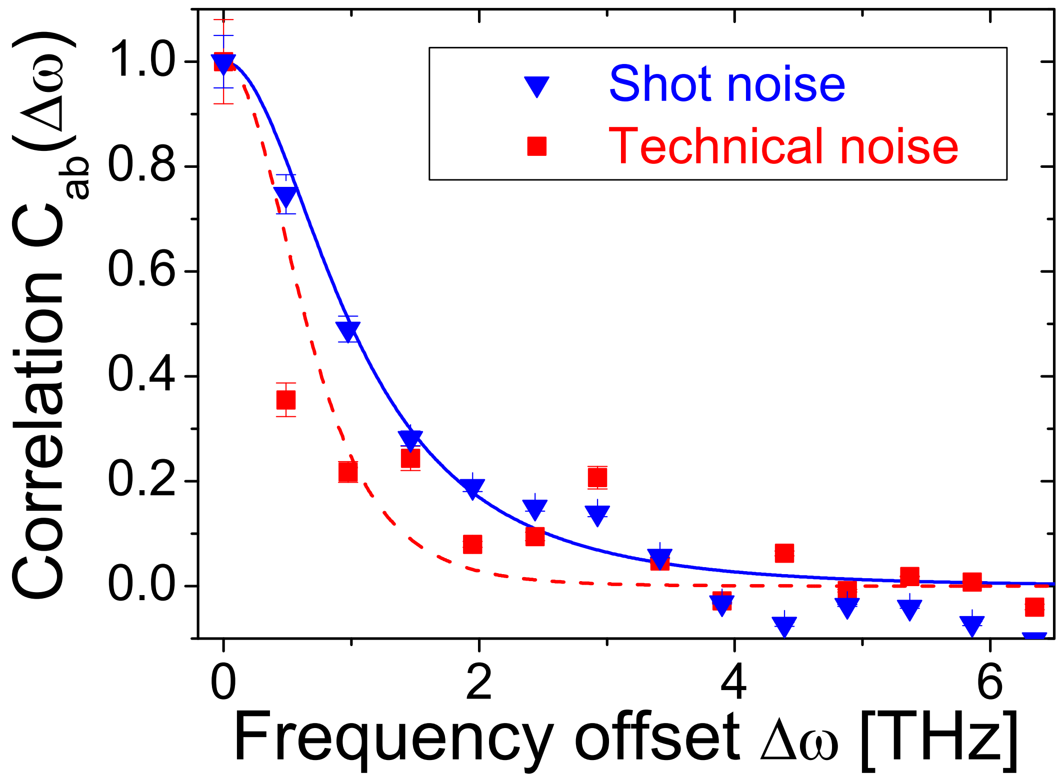

FIG. 5: Correlation function for shot noise (triangles) and technical noise (squares) as a function of frequency offset. The shot noise and technical noise data are compared to $\sqrt{C_{ab}^N}$ (full curve) and C_{ab}^N (dashed curve), respectively, as defined in Eq. (9). The measured correlation functions were normalized to unity to compensate for a non-perfect contrast due to the effect of stray intensity from other speckle spots.